

Bootstrap and the Parameters of Pion-Nucleon Resonances

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Abstract

In this talk we demonstrate the results of application of the perturbative effective theory formalism developed in papers [1] – [6] to the calculation of πN elastic scattering amplitude. Restrictions on the contributing resonance parameters are obtained and the low energy coefficients are calculated.

1 Introduction

In [1] – [6] it is shown that when working in effective theory formalism (in the sense of Weinberg), the assumption that the perturbation theory (loop expansion for the scattering amplitudes) is self consistent, together with the general requirements of covariance, unitarity, causality and crossing, leads to certain restrictions for the effective Hamiltonian parameters. Moreover, using concrete renormalization scheme, it is also possible to obtain constraints (the *bootstrap equations*) for the *physical* parameters of the given amplitude. In other words: one can obtain restrictions for the particle spectrum and, thus, perform a comparison with the experiment.

We are going to discuss how to obtain those restrictions in case of πN -elastic scattering. As an example, we make the accurate estimate of the tensor-to-vector ρNN coupling ratio in complete agreement with the experimental data which has never been explained in model-independent way. Besides, we present the values of the first 48 coefficients in the expansion of the tree amplitude around the crossing symmetry point.

The mathematical background for these calculations and the formalism used is reviewed in more details in the talk [7].

2 πN elastic scattering

The amplitude $M_{a\alpha}^{b\beta}$ of the reaction $\pi_a(k) + N_\alpha(p, \lambda) \rightarrow \pi_b(k') + N_\beta(p', \lambda')$ can be written in the following form:

$$M_{a\alpha}^{b\beta} = i(2\pi)^4 \delta(k + p - k' - p') \left\{ \delta_{ba} \delta_\alpha^\beta M^+ + i\varepsilon_{bac} (\sigma_c)^\beta_\alpha M^- \right\} .$$

Here

$$M^\pm = \bar{u}(p', \lambda') \left\{ A^\pm + \left(\frac{\not{k} + \not{k}'}{2} \right) B^\pm \right\} u(p, \lambda) ,$$

$\not{k} \equiv k_\mu \gamma^\mu$, $a, b = 1, 2, 3$ and $\alpha, \beta = 1, 2$ stand for the isospin indices, λ, λ' — for polarizations of the initial and final nucleons, respectively, $\bar{u}(p', \lambda')$, $u(p, \lambda)$ — for Dirac spinors, and σ_c , $c = 1, 2, 3$ — for Pauli matrices. The invariant amplitudes A^\pm and B^\pm are the functions of an arbitrary pair of scalar kinematical variables $s \equiv (p + k)^2$, $t \equiv (k - k')^2$, and $u \equiv (p - k')^2$.

To construct the *tree amplitude* one needs to write down the contributions of all possible contact vertices and resonance exchange graphs.

We work in the framework of effective theory formalism. This means that, when constructing the Hamiltonian, we need to take account of *all* the terms consistent with (algebraic) symmetry properties of strong interactions; there are no limitations on the number and order of field derivatives. Besides, in order to avoid model dependence we reserve the possibility to work with infinite number of resonance fields and unbounded (though, of course, discrete) mass spectrum.

Altogether this means that the number of items contributing to the tree level amplitude is actually infinite. This creates a problem: we have no guiding principle allowing to fix the order of summation. The way out of this difficulty has been pointed out in [1] – [6]. It consists of switching to the *minimal* parametrization for the Hamiltonian and using the method of Cauchy forms. The important advantage of this approach is that it results in uniformly converging series of pole terms defining the amplitude as the polynomially bounded meromorphic function — no kind of singularities but simple poles can appear on this way. To construct the Cauchy form for the tree amplitude under consideration, one needs to establish the residues (which are the function of coupling constants and masses) at the corresponding pole terms (masses) and to fix the bounding polynomial degree — it happens quite sufficient for fixing the amplitude up to few unknown functions which, in turn, can be found from the *bootstrap equations*

The origin of bootstrap equations is quite natural. Using the technique of Cauchy forms, we can get well defined uniformly convergent expansions for the invariant amplitudes (we do not write them down here due to the lack of space) in three different bands on the Mandelstam plane: $B_s\{s \sim 0\}$, $B_t\{t \sim 0\}$ and $B_u\{u \sim 0\}$. These bands obviously has non-empty intersections (near the corners of Mandelstam's triangle), and the corresponding Cauchy forms are different in each band. Since we need the tree amplitude to possess crossing symmetry, each invariant amplitude should be a meromorphic function on all the Mandelstam plane. Thus the relevant Cauchy forms should coincide in the band intersection domains. This results to the set of functional equations (bootstrap equations) for the tree level invariant amplitudes, or, the same, to infinite set of numerical equations for Hamiltonian parameters¹.

If one uses the renormalized perturbation theory and imposes the physical renormalization prescriptions, in which the tree amplitude is expressed in terms of *physical* parameters, then the bootstrap equations becomes the restrictions for the physical (measurable) spec-

¹It is interesting to note that in case of e.g. πN -elastic scattering some of those equations give explicit relations between bosonic and fermionic spectrum parameters, thus, demonstrating certain *supersymmetry* features.

trum. In other words, the obtained bootstrap equations remains true after renormalization.

It is these equations that can be tested substituting experimental data for resonance masses and widths. They also give a possibility to express one resonance parameter via the other, which, again, can be compared with the known data.

3 Calculation of G_T/G_V

The quantities $G_{NN\rho}^T$ and $G_{NN\rho}^V$ (our minimal parametrization couplings can be related to them) were defined and fitted in [8] as couplings in the following effective Hamiltonian:

$$H_{\text{eff}}^{NN\rho} = -\bar{N} \left[G_{NN\rho}^V \gamma_\mu \vec{\rho}^\mu - G_{NN\rho}^T \frac{\sigma_{\mu\nu}}{4m} (\partial^\mu \vec{\rho}^\nu - \partial^\nu \vec{\rho}^\mu) \right] \frac{1}{2} \vec{\sigma} N , \quad (1)$$

where σ_a are Pauli matrices and m is the proton mass.

The existing experimental data [8] give:

$$\frac{G_{NN\rho}^T}{G_{NN\rho}^V} \approx 6.1 , \quad \frac{G_{\pi\pi\rho} G_{NN\rho}^V}{4\pi} \approx 2.4 , \quad G_{\pi\pi\rho} \approx 6.0 . \quad (2)$$

Taking the relevant bootstrap equations (here - 2 of them) we treat the above couplings as unknown and express them via other resonance parameters², the resulting numerical equations being in complete agreement with (2) with 15% accuracy.

It should be noted, that the G_T/G_V ratio was recently calculated by the authors in the frame of KN -elastic scattering, again, in complete agreement with the experiment (to be published).

4 Low-energy coefficients

Using the Cauchy forms technique, we have calculated the coefficients in the expansion of the *tree* amplitude around the crossing symmetry point ($t, \nu_t \equiv s-u=0$). This coefficients certainly *will* be affected by loop corrections, however, as one can see from the Table 1, the tree level results are very close to the experimental values — this fact gives a hope that our way of constructing the tree amplitude [1]-[6] leads to nice convergence of loop expansion, at least, in low energy domain. In other words, the tree approximation gives nice description of the physical amplitude at low energies³.

Introducing the new quantity

$$C^\pm = A^\pm + \frac{m\nu_t}{4m^2-t} \tilde{B}^\pm ,$$

²These particular equations seems to converge fast: among the known resonances only $N(0.94)$, $N(1.44)$, $\Delta(1.23)$ and one meson — $\rho(0.77)$, give significant contributions, other possible contributions are suppressed by the inverse squares of their mass.

³It should be noted that, in all the cases we checked, the bootstrap equations are consistent with the experimental data only if the tree amplitude asymptotic is taken in accordance with the corresponding Regge intercept. In other words, the tree amplitude *shall* have the asymptotic close to the physical one.

B^+	b_{00}^+	b_{01}^+	b_{02}^+	b_{03}^+	b_{10}^+	b_{11}^+	b_{20}^+	b_{21}^+
Experiment	-3.50	+0.22	-0.10	-0.00036	-0.99	+0.095	-0.31	+0.42
	± 0.10	± 0.10	± 0.05	± 0.00004	± 0.01	± 0.015	± 0.01	± 0.08
Theory	-4.96	+0.18	-0.004	+0.0001	-1.00	+0.07	-0.19	+0.02
\tilde{B}^-	b_{00}^-	b_{01}^-	b_{02}^-	b_{03}^-	b_{10}^-	b_{11}^-	b_{20}^-	b_{21}^-
Experiment	+8.37	+0.19	+0.019	+0.0021	+1.08	-0.063	+0.30	-0.32
	± 0.23	± 0.07	± 0.007	± 0.0002	± 0.04	± 0.011	± 0.04	± 0.07
Theory	+8.56	-0.071	+0.002	+0.00003	+1.44	-0.063	+0.22	-0.018
A^+	a_{00}^+	a_{01}^+	a_{02}^+	a_{03}^+	a_{10}^+	a_{11}^+	a_{20}^+	a_{21}^+
Experiment	+25.5	+1.18	+0.035	+0.0060	+4.60	-0.051	+1.19	-0.056
	± 0.5	± 0.05	± 0.007	± 0.0005	± 0.12		± 0.07	
Theory	+30.2	+1.1	+0.04	+0.007	+6.28	-0.25	+1.23	-0.087
C^+	c_{00}^+	c_{01}^+	c_{02}^+	c_{03}^+	c_{10}^+	c_{11}^+	c_{20}^+	c_{21}^+
Experiment	+25.5	+1.18	+0.035	+0.0060	+1.12	+0.15	+0.20	+0.034
	± 0.5	± 0.05	± 0.007	± 0.0005	± 0.02	± 0.01	± 0.01	± 0.010
Theory	+30.2	+1.1	+0.04	+0.007	+1.3	-0.10	+0.22	-0.023
A^-	a_{00}^-	a_{01}^-	a_{02}^-	a_{03}^-	a_{10}^-	a_{11}^-	a_{20}^-	a_{21}^-
Experiment	-8.87	-0.34	+0.1	-0.0021	-1.25	+0.023	-0.338	+0.305
Theory	-9.85	+0.2	-0.004	+0.00007	-1.55	+0.08	-0.27	+0.023
C^-	c_{00}^-	c_{01}^-	c_{02}^-	c_{03}^-	c_{10}^-	c_{11}^-	c_{20}^-	c_{21}^-
Experiment	-0.50	-0.10	+0.12	+0.00032	-0.17	-0.039	-0.038	-0.013
	± 0.05	± 0.01	± 0.04	± 0.00003	± 0.01	± 0.005	± 0.004	± 0.004
Theory	-0.6	+0.09	-0.0019	+0.0001	-0.18	+0.026	-0.035	+0.006

Table 1: Low energy coefficients (calculated at the tree level) and their experimental values (averaged). In the case of A^- it is meaningless to calculate errors: the corresponding quantities are too sensitive to the uncertainties in experimental data.

where \tilde{B}^\pm is just B^\pm with the nucleon pole subtracted⁴, we define the low-energy coefficient b_{mn}^\pm , a_{mn}^\pm , and c_{mn}^\pm in the following way:

$$\begin{aligned}\tilde{B}^+(t, \nu_t) &= \nu_t \sum_{m,n} b_{mn}^+(\nu_t^2)^m t^n, & \tilde{B}^-(t, \nu_t) &= \sum_{m,n} b_{mn}^-(\nu_t^2)^m t^n, \\ \tilde{A}^+(t, \nu_t) &= \sum_{m,n} a_{mn}^+(\nu_t^2)^m t^n, & \tilde{A}^-(t, \nu_t) &= \nu_t \sum_{m,n} a_{mn}^-(\nu_t^2)^m t^n, \\ \tilde{C}^+(t, \nu_t) &= \sum_{m,n} c_{mn}^+(\nu_t^2)^m t^n, & \tilde{C}^-(t, \nu_t) &= \nu_t \sum_{m,n} c_{mn}^-(\nu_t^2)^m t^n,\end{aligned}$$

where all the expansions are around the point $t, \nu_t = 0$. Re-expanding corresponding Cauchy forms around this point in the above (Taylor) form, using experimental data for couplings and masses and neglecting all the contributions of the resonances with $M \geq 1.9$ GeV, we get numerical values for the coefficients⁵ (see Table 1).

Actually, among baryons only $\Delta(1.23)$ and $N(1.44)$ give non-negligible contributions as well as σ among mesons, all other known resonances give less than 10%.

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⁴That is what can be compared with the experiment: the nucleon pole contribution is dominant in this momentum region but can be excluded in experimental data analysis. On the other hand, in our formulas we have this contribution explicitly and can simply remove it by hand.

⁵The experimental data can be found in [8]: please note, that they use somewhat different definitions for low-energy coefficient, so one needs to perform certain recalculations to compare the results.

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